Dark Matter, A Newtonian Approach

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***Abstract*—In this paper I have tried to present a case for dark matter using a Newtonian approach. I have also tried to present the evidence for the same.**

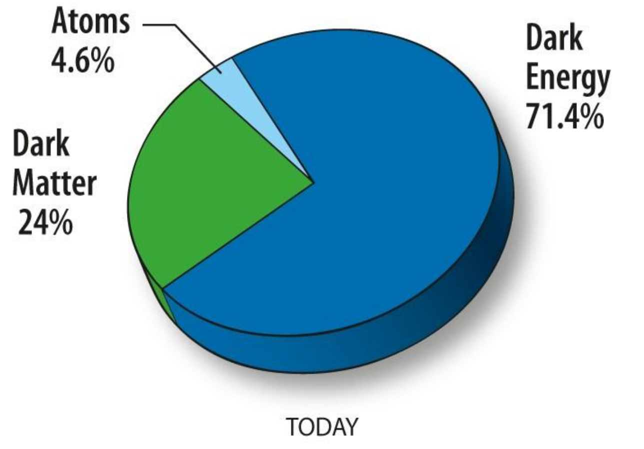
In the first three sections of the paper, I have established a relationship between gravitation and velocity using Newton's laws and then showed how the rotation velocities of the galaxies in the Coma cluster do not follow the predicted values of the velocities and finally how inclusion of extra mass can explain the apparent disagreement of the observed phenomenon with the predicted values.

In the next two sections I have tried to provide some evidence for the presence of dark matter in our universe.

Keywords—Dark matter, Universe, Gravitational lensing, Spacetime

I. Introduction

Our entire universe isn't what we see it to be or think it to be. Everything we see in the universe is made up of known visible matter but still, that only accounts for only about 4.6% of the total matter energy density of the universe. Then one question arises in our mind? Where's the rest of it? recent studies have found that our universe is only 4.6% visible matter or normal matter as We know it, about 24% of the entire universe is dark matter and the remaining 71.4% consists of dark energy.



Dark matter was discovered by Dr. Fritz Zwicky in the year 1933 while he was observing the Coma cluster of galaxies. Dr. Zwicky was analyzing the relative velocities of the galaxies under the influence of gravity of the system. Before we go to the topic of how he managed to do that let us first take a look at some basic principles of gravity to better understand Zwicky's results and how did he arrive at his conclusion. It is important to note that, although here I'm using a Newtonian approach to explain dark matter (a relatively simpler approach), Dr. Zwicky used something called the "virial theorem" that gives a general equation for the average kinetic energy of particles bounded by potential forces.

II. Gravitation and Velocity

According to Newton's universal law of gravitation.

$$F = \frac{GMm}{R^2} \quad \text{............ (1)}$$

We know in stable orbit conditions;

$$F = \frac{mv^2}{R} \quad \text{..............(2)}$$

Where ***F*** is the force of gravity, ***M*** is the center of mass of the gravitationally bounded system, ***m*** is the mass of the individual body under consideration of which the orbital velocity ***v*** due to gravity is being measured.

Equating equations 1 and 2 we get,

$$\frac{mv^2}{R} = \frac{GMm}{R^2}$$

So, we get,

$$v = \sqrt{\frac{GM}{R}} \quad \text{. ....(3)}$$

As we can see velocity due to gravity is a function of distance from the gravitational center of mass. Equation (3) tells us that if there is a gravitationally bounded system where a body with mass ***m*** orbits around the center of mass of the system with mass ***M***, then the orbital velocities of the body is a function of distance from the center mass of the system denoted by ***R***. Now according to equation (3),

$$v = \sqrt{\frac{GM}{R}}$$

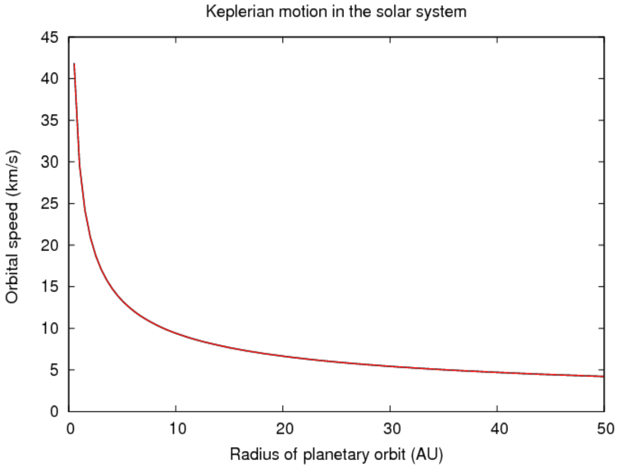
but as we can see both ***G*** and ***M*** are constants in the system, we can write the equation as follows,

$$v = \frac{K}{\sqrt{R}}$$

or we can say that,

$$v \propto \frac{1}{\sqrt{R}}$$

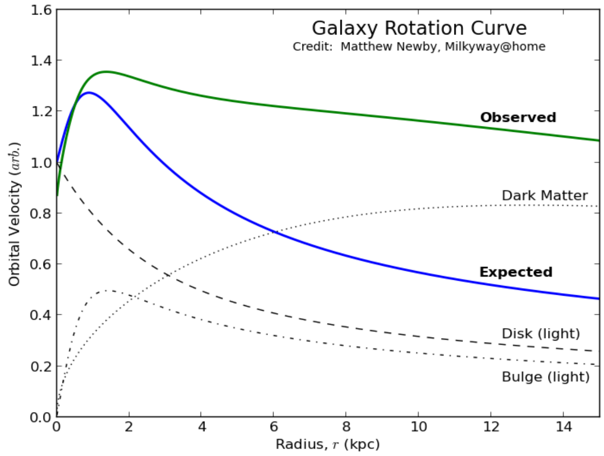
from the above expression we can say that as we move farther away from the gravitational center of mass, we would see the orbital velocities of the bodies in the system decreasing in accordance with the above relation. Given below is a classic example of the above derived result. It shows how the orbital velocities of planets decrease as we move farther away from the gravitational center of mass of the solar system, in other words, The Sun.

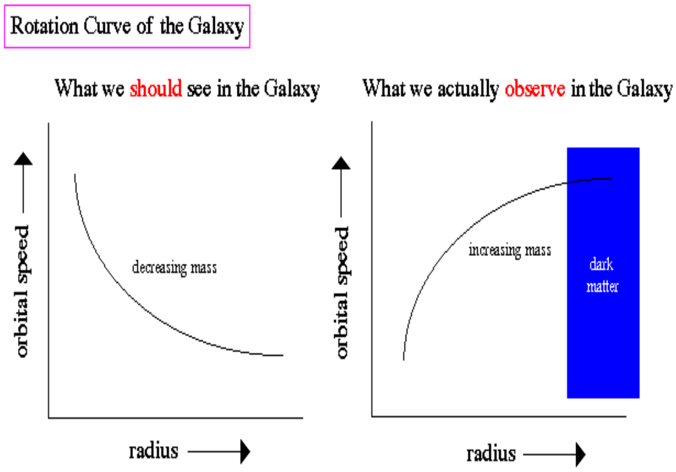


Precisely this is what Dr. Zwicky intended to find. Imagine a galaxy, in it there are trillions of stars orbiting around its galactic center. Now we can assume that as we move farther and farther away from the galactic center the orbital velocities of the stars will gradually decrease. So, what did Dr. Zwicky find?

III. Dr. Zwicky's Results and How It Leads to Dark Matter

Dr. Zwicky used virial theorem to predict the orbital velocities of galaxies of the Coma cluster around the common center of mass of the system. We can do the same thing by the Newtonian approach. We can take the help of the expression for the orbital velocities of gravitationally bounded objects derived in the earlier section of this paper and predict the velocity distribution of the stars in those galaxies with respect to their distance from their respective galactic centers. Then we can plot the predicted results on a graph and intend it to match with the results obtained from the observations made. Did Dr. Zwick's observations match the predictions that he made or the predictions that we made with our Newtonian approach? The answer is no! The analysis of the observed data showed that the velocities of the stars orbiting at the outer edge of the galaxies were roughly equal in magnitude with the orbital velocities of the stars orbiting near the galactic center. Dr. Zwicky's observed results were way off from his predicted plot. These results force us to ask the question, "is there something wrong with Newtonian physics? Or is there a deeper reality hidden behind it waiting to be discovered?" There is a little chance that the former is right because Newtonian mechanics lays the groundwork for the entire classical physics and it is among one of the most successful ways to explain the universe in the macroscopic scales. We have that confidence on Newtonian mechanics that it cannot be wrong. This leaves room for the later statement to be true in some sense, so we can insist on the later statement, there must be some deeper mystery hidden inside waiting to be discovered.





So why were the stars having velocities they shouldn't have? To answer this, we need to go back to equation (3). Imagine a star orbiting the galactic center at a distance ***R***, so it should have a velocity equal to,

$$v = \sqrt{\frac{GM}{R}}$$

but instead, we find it to have a velocity

$u > v$.

But we know ***G*** cannot change, since it is the universal gravitational constant and we know that the star is at a distance ***R***, so the only parameter that needs to change here in order to account for the extra velocity is ***M***, in other words the galactic system must have some hidden extra mass that is responsible for the extra orbital velocities of the stars. And since the effect is spread throughout the galaxy, the extra mass must be distributed roughly uniformly throughout the entire galaxy. So how much extra mass is present in the galaxy apart from the visible matter or known form of matter?

Let the extra mass of the galaxy due to the mysterious form of matter be ***m***, so according to the expression of velocity we derived earlier,

$$u = \sqrt{\frac{G(M+m)}{R}} \quad ; \ldots\ldots. (4)$$

$$u^2 = \frac{G(M+m)}{R} \; ;$$

$$u^2 = \frac{GM}{R} + \frac{Gm}{R} \; ;$$

$$u^2 = v^2 + \frac{Gm}{R} \; ; \text{[from equation (3)]}$$

$$u^2 - v^2 = \frac{Gm}{R} \; ;$$

$$m = \frac{R\,(u^2 - v^2)}{G} \quad \ldots\ldots\ldots\ldots\ldots(5)$$

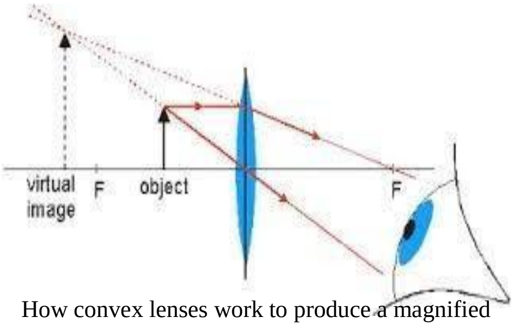
The result obtained above is a very crucial one, it tells us how much of extra unknown form of matter must be present in the galaxies in order for the stars to have the extra orbital velocities around the galactic centers. And as we can see that ***G*** is very small and *R* usually is large, so the value of ***m*** must be significantly large. Dr. Zwicky called this significantly large extra unknown mass, "The Dark Matter". This extra mass does not come from the ordinary matter as we know it, rather it comes from a totally unknown form of matter which we don't know much about. It is important to note that the word "Dark" in the name doesn't have anything to do with color, it represents our ignorance, that how little we know about this form of matter.

IV. Other Evidences for Dark Matter

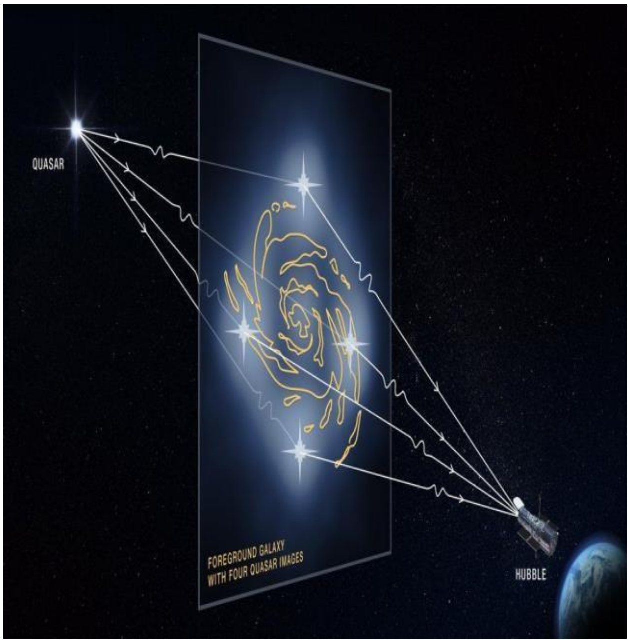
We don't know what dark matter is or where does it come form, but one thing we certainly know is that it's out there, its present in the interstellar space it's present in the intergalactic space, basically dark matter is present everywhere. So, what are the evidences?

A. *Gravitational Lensing:*

According to general theory of relativity matter and energy can bend the fabric of spacetime, more massive the body or the matter energy density more it bends the fabric of spacetime. Since everything in this universe is confined within the fabric of spacetime they are bound to follow the curvature wherever it leads and this eventually gives rise to a force called gravity. So, if we ask," what causes gravity?'' Einstein would answer in this fashion, "matter tells spacetime to bend and curve and spacetime tells matter to follow the curvature''. One interesting consequence of this result is that light is also made up of particles called photons, and since photons are particles, they are bound to follow the curvature of spacetime. In other words, gravity will force light to bend around massive objects, the same light ray that prefers to travel in a straight line otherwise. Now we know how convex lenses work and how it magnifies an image. Light rays while passing through a lens bends due to the difference in refractive index and that eventually creates the effect of magnification. Similarly light rays while passing from near a massive object ends up bending in a similar way creating a lensing effect. And this is called gravitational lensing.



How convex lenses work to produce a magnified virtual image

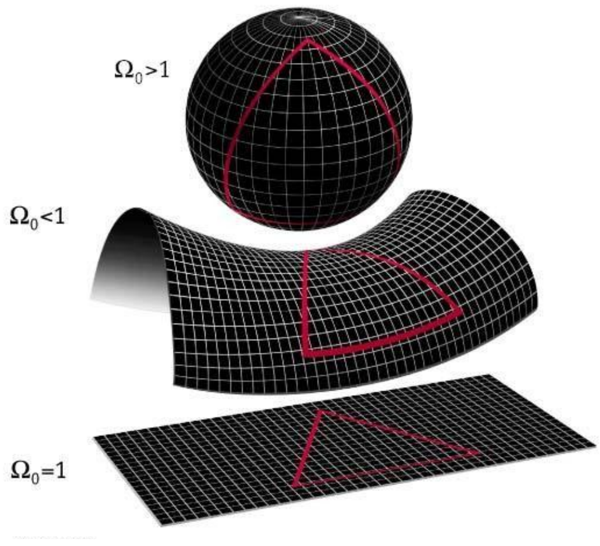


How gravitational lensing works to produce a magnified virtual image of the distance quasar

What does this have to do anything with dark matter? Well just as general theory of relativity leads us to gravitational lensing it also tells us by how much amount a light ray should bend around an object of given mass. So, by observing the light coming from a distant galaxy we can calculate its mass and then we can apply the mathematics of general relativity to calculate by how much the light coming from the background galaxies should bend around the galaxy under observation. But when we make an observation of the light coming from the background galaxies, we find that the degree of lensing is higher than expected and the mass of the foreground galaxy is not enough to account for the extra lensing. Which quite inevitably leads us to the conclusion that there is some extra unknown form of matter present in the galaxy the nature of which is still unknown to us which is responsible for the extra lensing.

B. Geometry of the universe:

what do we mean when we say geometry of the universe? Imagine the fabric of spacetime as a giant curtain. As we know from the results of general relativity that matter bends spacetime, so if the universe has too much matter in it the space shall bend too much until it closes in on itself. Just like the giant curtain, if you put some weight on it will bend inward, if you keep on adding more and more weight eventually the curtain will get so much bent that it's two opposite ends will come together and form a closed structure this is called positive curvature. But if there is too less matter present in the universe the expansion of the universe will cause the fabric of the universe to bend in the opposite direction which will never really close but will go on bending forever, imagine the curtain being stretched from all sides and you don't add enough weight to keep it bending inward, this is called negative curvature which will cause the universe to have a horse saddle like structure. What if there is just enough matter that makes the fabric of spacetime bend inward counteracting the tendency of the fabric to bend outward due to cosmic expansion? In other wards if there's just enough matter present it can stop the universe from bending in either way and can result in a flat universe. Now this is not something our mind can grasp. How can three-dimensional space be flat or bend in a certain way? But we are just three-dimensional beings stuck with our three-dimensional imaginations. Imagine you were a two dimensional being living in a piece of paper. Only spatial dimensions you are aware of are back and forth and left and right. You would have no idea about a third dimensions top and bottom and most certainly you would not have a shred of imagination of how the paper can be bent in certain ways, for example like a bow.



Three possible geometries of our universe

Now the question arises, which kind of universe do we live in? The analysis of the data obtained from the Cosmic Microwave Background Radiation (CMBR) shows us that we live in a flat universe. That is in our universe there is just enough matter to make it flat. But there's a problem with this picture, when we set out to find how much matter is present in the universe, we find that the total amount of known matter that accounts for all the stars planets and galaxies in the universe only represents 5% of the total matter required to make the universe flat, how can this be possible? We know that the universe is flat but still the other 95% of matter in the universe is missing. Discovery of dark matter turned up to be a major clue to solve that problem and a missing piece of this great puzzle.

If dark matter wasn't there, gases could never have collapsed to form galaxies and galactic clusters. Dark matter provided that foundation for our universe to exist the way it exists today.

V. PROPERTIES OF DARK MATTER

We still don't have a clear understanding of what dark matter is or where does it come from, but still, we can make educated guesses about some properties that dark matter must have or else the universe wouldn't look the way it is now. Some of these properties are,

- It is nothing like the matter as we know it.
- It does not interact with light or else we would have detected it by now.
- It does not interact with ordinary matter in any known way other than gravity.
- It bends the fabric of spacetime around its vicinity just like any matter with mass would do according to the laws of general relativity, as a result it shows gravitational effects.
- It does not clump together.

But as of now still the concept of dark matter remains a mystery to us till date. Huge efforts are being made to detect dark matter in our solar system, but till now all such attempts have been proved to be fruitless. All we can say is our solar system doesn't have enough dark matter to show significant gravitational anomaly or the effects of dark matter is only evident in large scale structures like galaxies and galaxy clusters. We simply don't know enough about dark matter to setup and design experiments to detect it yet. But we certainly know enough to tell that it is out there. The very goal of science is to chart unknown territories of knowledge, we might not know enough for now but who knows when someone new will come up with a new breakthrough and turn our scientific understanding upside down all over again.

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REFERENCES

[1] Dr. Lawrence M Krauss, "A Universe From Nothing, why there is something rather than nothing", Simon & Schuster 2012, pp. 25,31,34-35,40-41,72,87

[2] Dr. Stephen W Hawking, “A Brief History Of Time”, A Bantam Book : 9780553175219, pp. 152-157

[3] Dr. Stephen W Hawking, “The Theory of Everything, the origin and fate of the universe lecture series”, Jaico Books,Special anneversary edition, pp. 19,36

[4] Dr, Brian Greene, “ The Hidden Reality”, Penguine Books, pp. 24-25

[5] https://www.nature.com/articles/d41586-019-02603-

[6] Lawrence M. Krauss, “A Universe from Nothing”, Radcliffe Institute lecture https://youtu.be/vwzbU0bGOdc